THE AVERAGE APPROXIMATING METHOD ON FUNCTIONAL ADJUSTMENT QUANTITY FOR SOLVING

The Volterra Integral Equation II

(**corrected for solving integral equations with Hereditary kernels**)

by Co.H Tran , University of Natural Sciences , HCMC Vietnam -
Institute of Applied Mechanics , HCMC - coth123@math.com & coth123@yahoo.com

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Sat , November 06 2004

** ***Abstract*** : Solving the Volterra's integral equation II with applying the Neumann series and the average approximating method on functional adjustment quantity .
** ***Subjects:*** Viscoelasticity Mechanics , The Integral equation .

The Average Approximating Method on Functional Adjustment Quantity (Sokolov's method)

In consideration of The Volterra Integral Equation II (second kind) , we find the explicit expression for the resolvent kernel Γ (t , t) in the general form :

$$v = (1 + \lambda K^*)\, u$$

here λ : arbitrary parameter . The solution of u can be represented with the Neumann series : .

The resolvent operator Γ^* is determined by a Neumann series : , then the kernel .

The convergence of this series must be investigated in a connection with the Neumann series .
The average approximating method on the functional adjustment quantity (Sokolov's method) makes increasing for the rate of convergence of this series .
From the first approximation of the solution u , we find the adjustment quantity for the next and so on .
We consider the following equation :
(1)
the first approximation : (2) by choosing the initial adjustment quantity : (3)
From (2) and (3) we obtain : (4) with (5)
the n-th approximation : (6) and the adjustment quantity of the n-th order can then be written as :
(7) here (8) . From (6) , (7) and (8) we have : (9)
Denoting the formulas (6) to (9) can be carried out by the computer programming language. We can show that the convergence condition of this method is (10) here : the project-operator from the Banach's space B into its space Bo (the solution u B)

Sokolov's method

As seen , the first approximation : $u1(t) = v(t) + \lambda\,\alpha_1 \int_a^b K(t,\tau)\,d\tau$ We

choose the initial adjustment quantity : $\alpha_1 = \dfrac{\int_a^b u1(t)\,dt}{b-a}$ with $u_0(x) := 0$;

$T_1(x) := u_1(x)$

The adjustment quantity of the order i-th can be expresssed :

$$\alpha_{i+1} := \frac{\lambda \int_a^b K\,(T_i(y) - T_{i-1}(y) - \alpha_i)\,dy\,(b-a)}{D}$$

The coefficient

$$\alpha_{i+2} := \min\left(\alpha_i, \frac{\lambda \int_a^b K\,(T_i(y) - T_{i-1}(y) - \alpha_i)\,\mathrm{d}y\,(b-a)}{\mathrm{D}}\right)$$

Compare with the initial function and we have the error estimated :

$$saiso_i := \sqrt{\frac{\int_a^b |u_{i+1}(x) - u_i(x)|^2\,\mathrm{d}x}{b-a}}$$

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> restart; interface(warnlevel=0):
> xapxi:=proc(f,lambda,a,b,n,K)
> local s,smax,smin,saiso,i,D,T,eq1,alpha,eq,alpha1,iv,dv;
eq1:=u(x)=f+lambda*int(K*u(y), y=a..b);;printf("\n%s","
");print("*****************************************************
********************");print("                          THE
VOLTERRA INTEGRAL EQUATION II ( second kind )
:",eq1,"*****************************************************
******************");u[1](x):=f+alpha1*lambda*int(K,
y=a..b);alpha:=int(u[1](x),
x=a..b);eq:=alpha1=alpha;printf("\n%s"," RECURRING TASKS
NUMBER ");print("1&/.THE EQUATION OF 1st ADJUSTMENT
QUANTITY : ", eq);alpha1:=Re(solve(eq,alpha1));print(" THE
1st ADJUSTMENT QUANTITY :
alpha[1]=",evalf(alpha1,3));alpha[1]:=alpha1;u[1](x):=f+alp
ha[1]*lambda*int(K, y=a..b);print(" FUNCTION
u[1](x)=",u[1](x));;
> T[0](x):=0:
> T[1](x):=x->u[1](x);D:=b-a-lambda*int(int(K, y=a..b),
x=a..b);
for  i  from  1  to  n  do
u[0](x):=0:T[0](y):=0:T[1](x):=u[1](x);T[i](y):=subs(x=y,T[
i](x));T[i-1](y):=subs(x=y,T[i-1](x));;printf("\n%s","
");print(" COMPARE WITH THE INITIAL FUNCTION",u[i-
1],"(x)=",T[i-1](x));;printf("\n%s"," ");print(" ----------
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----------------------");printf("\n%s"," RECURRING LOOP No
");printf("\n%s"," "); ;print(i+1,"&/.  ADJUSTMENT QUANTITY
OF :",i+1," order WE OBTAIN :
");;alpha[i+1]:=(lambda/D)*int(int(K*(T[i](y)-T[i-1](y)-
alpha[i]), y=a..b), x=a..b);
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alpha[i+1]:=evalf(alpha[i+1],5);;alpha[i+2]:=min(alpha[i+1]
,alpha[i]);;alpha[i+2]:= evalf(alpha[i+2],5);
;T[i+1](x):=f+lambda*int(K*(T[i](y)+alpha[i+1]),
y=a..b);u[i+1](x):=T[i+1](x);;print(" FUNCTION
",u[i+1],"(x)=",T[i+1](x));print(" ALPHA
COEFFICIENT",[i+1],"="
,evalf(alpha[i+1],5));;;;saiso[i]:=sqrt((1/(b-
a))*int((abs(u[i+1](x)-u[i](x))^2), x=a..b));;;;print("
ERROR OF : ",u[i+1],"(x)"," AND ",u[i],"(x)"," IS
:");print( " ERROR ESTIMATED = ",evalf(saiso[i],5));
> od:printf("\n%s"," ");printf("\n%s"," "); print("--------
------------------------CONCLUSION-------------------------
------- ");;printf("\n%s"," ");print( "THE ESTIMATED ERRORS
OF ");for i from 1 to n do print( " order ",[i]," IS
:", evalf(saiso[i],5));od:;printf("\n%s","
");printf("\n%s"," ");
> end:
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It is easy to see that $u^k(x)$ n is a Cauchy sequence in L2(T) as k -> ∞.

It follows from the completeness of L2(T) that it converges in the L2 sense to a sum g in L2(T). That is, we have

lim || $u^k(x)$ - $u^{(k-1)}(x)$ || = 0 k -> ∞

```
> dothi:=proc(k,m,n,h)
> local y,ym,yn;
>
with(plottools):with(plots):y:=u[k](x);ym:=u[m](x);yn:=u[n]
(x);print("GRAPHIC :", u[k],"(x) = ",u[k](x)," RED
");;print("GRAPHIC :", u[m],"(x) = ",u[m](x)," YELLOW
");;print("GRAPHIC :", u[n],"(x) = ",u[n](x)," BLUE ");
:plot([y(x),ym,yn],x=-
h..h,color=[red,yellow,blue],thickness=[3,9,15],title='APPR
OXIMATEDGRAPHICS');
> end:
> xapxi(-20.2*sqrt(x), 3 , 0 , 1 , 10 , sqrt(x)*(y+10)
);
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"   THE VOLTERRA INTEGRAL EQUATION II (second kind) :", $u(x) = -20.2\sqrt{x} + 3 \int_0^1 \sqrt{x}\ (y+10)\ u(y)\ dy$

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RECURRING TASKS NUMBER

"1&/.THE EQUATION OF 1st ADJUSTMENT QUANTITY ; $\alpha 1 = -13.46666667 + 21.\ \alpha 1$

" THE 1st ADJUSTMENT QUANTITY : alpha[1,]=", 0.673

" FUNCTION u[1](x)=", $1.01000000\sqrt{x}$

" COMPARE WITH THE INITIAL FUNCTION", u_0, "(x)=", 0

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RECURRING LOOP No

2, "&/. ADJUSTMENT QUANTITY OF ;", 2, " order WE OBTAIN : "

" FUNCTION ", u_2, "(x)=", $0.99990105\sqrt{x}$

" ALPHA COEFFICIENT", [2], "=", -0.0067333

" ERROR OF : ", u_2, "(x)", " AND ", u_1, "(x)", " IS :"

" ERROR ESTIMATED =", 0.0071410

" COMPARE WITH THE INITIAL FUNCTION", u_1, "(x)=", $1.01000000\sqrt{x}$

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RECURRING LOOP No

3, "&/. ADJUSTMENT QUANTITY OF ;"3, " order WE OBTAIN : "

" FUNCTION ", u_3, "(x)=", $1.00000098\sqrt{x}$

" ALPHA COEFFICIENT", "[3], "=", 0.000066626

" ERROR OF : ", u_3, "(x)", " AND ", u_2, "(x)", " IS :"

" ERROR ESTIMATED =", 0.000070661

" COMPARE WITH THE INITIAL FUNCTION", u_2, "(x)=", $0.99990105\sqrt{x}$

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RECURRING LOOP No

4, "&/. ADJUSTMENT QUANTITY OF :",4, " order WE OBTAIN : "

" FUNCTION ", u_4, "(x)=", $0.99999999\sqrt{x}$

" ALPHA COEFFICIENT", [4], "=", $-6.5990\ 10^{-7}$

" ERROR OF : ", u_4, "(x)", " AND ", u_3, "(x)", " IS :"

" ERROR ESTIMATED =,"$7.0004\ 10^{-7}$

" COMPARE WITH THE INITIAL FUNCTION", u_3, "(x)=", $1.00000098\sqrt{x}$

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RECURRING LOOP No

5, "&/. ADJUSTMENT QUANTITY OF :",5, " order WE OBTAIN : "

" FUNCTION ", u_5, "(x)=", $1.00000000\sqrt{x}$

" ALPHA COEFFICIENT", [5], "=", $6.7050\ 10^{-9}$

" ERROR OF : ", u_5, "(x)", " AND ", u_4, "(x)", " IS :"

" ERROR ESTIMATED =,"$7.0711\ 10^{-9}$

" COMPARE WITH THE INITIAL FUNCTION", u_4,
"(x)=", $0.99999999\sqrt{x}$

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RECURRING LOOP No

6, "&/. ADJUSTMENT QUANTITY OF ;", 6, " order WE OBTAIN : "

" FUNCTION ", u_6, "(x)=", $1.00000000\sqrt{x}$

" ALPHA COEFFICIENT", [6], "=", $-2.6417\ 10^{-11}$

" ERROR OF : ", u_6, "(x)", " AND ", u_5, "(x)", " IS :"

" ERROR ESTIMATED =, "0.

" COMPARE WITH THE INITIAL FUNCTION", u_5,
"(x)=", $1.00000000\sqrt{x}$

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RECURRING LOOP No

7, "&/. ADJUSTMENT QUANTITY OF ;"7, " order WE OBTAIN : "

" FUNCTION ", u_7, "(x)=", $1.00000000\sqrt{x}$

" ALPHA COEFFICIENT", [7], "=", $-2.7738\ 10^{-11}$

" ERROR OF : ", u_7, "(x)", " AND ", u_6, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_6, "(x)=", $1.00000000\sqrt{x}$

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RECURRING LOOP No

8, "&/. ADJUSTMENT QUANTITY OF ;"8, " order WE OBTAIN : "

" FUNCTION ", u_8, "(x)=", $1.00000000\sqrt{x}$

" ALPHA COEFFICIENT," [8], "=", $-2.9125\ 10^{-11}$

" ERROR OF : ", u_8, "(x)", " AND ", u_7, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_7, "(x)=", $1.00000000\sqrt{x}$

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RECURRING LOOP No

9, "&/. ADJUSTMENT QUANTITY OF ;"9, " order WE OBTAIN : "

" FUNCTION ", u_9, "(x)=", $1.00000000\sqrt{x}$

" ALPHA COEFFICIENT," [9], "=", $-3.0581\ 10^{-11}$

" ERROR OF : ", u_9, "(x)", " AND ", u_8, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_8, "(x)=", $1.00000000\sqrt{x}$

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RECURRING LOOP No

10, "&/. ADJUSTMENT QUANTITY OF ;"10, " order WE OBTAIN : "

$$\text{" FUNCTION "}, u_{10}, \text{"(x)="}, 1.00000000 \sqrt{x}$$

$$\text{" ALPHA COEFFICIENT"}, [10], \text{"="}, -3.2110\ 10^{-11}$$

$$\text{" ERROR OF : "}, u_{10}, \text{"(x)"}, \text{" AND "}, u_9, \text{"(x)"}, \text{" IS :"}$$

" ERROR ESTIMATED =", 0.

$$\text{" COMPARE WITH THE INITIAL FUNCTION"}, u_9, \text{"(x)="}, 1.00000000 \sqrt{x}$$

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RECURRING LOOP No

11, "&/. ADJUSTMENT QUANTITY OF ;"11, " order WE OBTAIN : "

$$\text{" FUNCTION "}, u_{11}, \text{"(x)="}, 1.00000000 \sqrt{x}$$

" ALPHA COEFFICIENT", [11], "=", $-3.3716\ 10^{-11}$

" ERROR OF : ", u_{11}, "(x)", " AND ", u_{10}, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

"-------------------------------CONCLUSION--------------------- \
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-------------------------------CONCLUSION------------------------------- "

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THE ESTIMATED ERRORS OF '

" order ", [1], " IS :", 0.0071410

" order ", [2], " IS :", 0.000070661

" order ", [3], " IS :", $7.0004\ 10^{-7}$

" order ", [4], " IS :", $7.0711\ 10^{-9}$

" order ", [5], " IS :", 0.

" order ", [6], " IS :", 0.

" order ", [7], " IS :", 0.

" order ", [8], " IS :", 0.

" order ", [9], " IS :", 0.

" order ", [10], " IS :", 0.

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> dothi(2,1,8,15);
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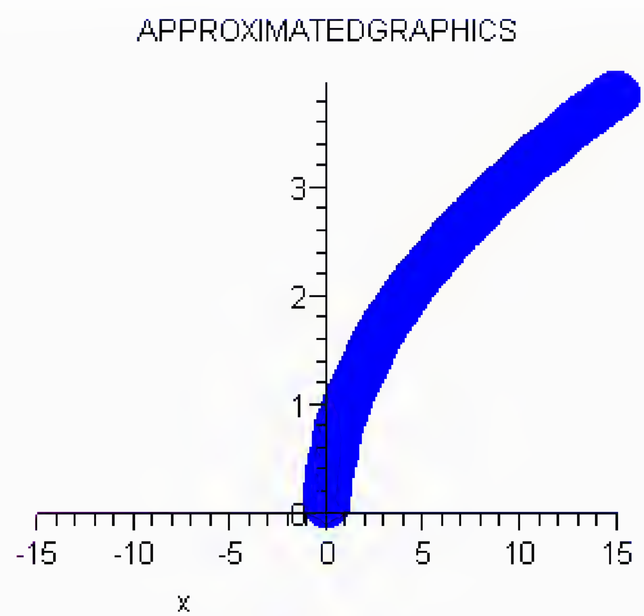
$$\text{"GRAPHIC :"}, u_2, \text{"(x) = "}, 0.99990105\sqrt{x}, \text{"RED"}$$

$$\text{"GRAPHIC :"}, u_1, \text{"(x) = "}, 1.01000000\sqrt{x}, \text{"YELLOW"}$$

$$\text{"GRAPHIC :"}, u_8, \text{"(x) = "}, 1.00000000\sqrt{x}, \text{"BLUE"}$$

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> 1+Int(x*y*u(y),y = 0 .. x);
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$$1+\int_0^x x\,y\,u(y)\,\mathrm{d}y$$

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> K:=1/18*x^7*y+1/18*x*y^7-1/9*x^4*y^4;
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$$\frac{1}{18}x^7y+\frac{1}{18}xy^7-\frac{1}{9}x^4y^4$$

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> xapxi(   1   ,   1   ,  0    ,  1    ,   15    ,  K  );
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"   THE VOLTERRA INTEGRAL EQUATION II (
    second kind) :," $u(x) = 1 +$

$$\int_0^1 \left(\frac{1}{18}x^7 y + \frac{1}{18}x y^7 - \frac{1}{9}x^4 y^4\right) u(y)\, \mathrm{d}y$$

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RECURRING TASKS NUMBER

"1&/.THE EQUATION OF 1st ADJUSTMENT QUANTITY ; "$\alpha 1$

$$=1 + \frac{1}{400}\,\alpha 1$$

" THE 1st ADJUSTMENT QUANTITY : alpha[1,]=",00

" FUNCTION u[1](x)=," $1 + \frac{100}{3591}x^7 + \frac{25}{3591}x - \frac{80}{3591}x^4$

" COMPARE WITH THE INITIAL FUNCTION"u_0, "(x)=", 0

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RECURRING LOOP No

2, "&/. ADJUSTMENT QUANTITY OF ;"2, " order WE OBTAIN : "

" FUNCTION ", u_2, "(x)=", 1

$$+ 0.006987451495\,x - 0.02233406016\,x^4 + 0.02787245210\,x^7$$

" ALPHA COEFFICIENT,"[2], "=", 0.0000047046

" ERROR OF : ", u_2, "(x)", " AND ", u_1, "(x)", " IS :"

" ERROR ESTIMATED =, "0.0000062884

" COMPARE WITH THE INITIAL FUNCTION,"u_1, "(x)=", 1

$$+ \frac{100}{3591}\,x^7 + \frac{25}{3591}\,x - \frac{80}{3591}\,x^4$$

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RECURRING LOOP No

3, "&/. ADJUSTMENT QUANTITY OF ;"3, " order WE OBTAIN : "

" FUNCTION ", u_3, "(x)=", 1

$$+ 0.006987409702\,x - 0.02233396848\,x^4 + 0.02787243020\,x^7$$

" ALPHA COEFFICIENT", [3], "=", $-5.2971\ 10^{-9}$

" ERROR OF : ," u_3, "(x)", " AND ", u_2, "(x)", " IS :"

" ERROR ESTIMATED =,"1.1818 10^{-8}

" COMPARE WITH THE INITIAL FUNCTION", u_2, "(x)=", $1 + 0.006987451495\,x - 0.02233406016\,x^4 + 0.02787245210\,x^7$

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RECURRING LOOP No

4, "&/. ADJUSTMENT QUANTITY OF ;"4, " order WE OBTAIN : "

" FUNCTION ", u_4, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [4], "=", 1.7406 10^{-11}

" ERROR OF : ," u_4, "(x)", " AND ", u_3, "(x)", " IS :"

" ERROR ESTIMATED =,"3.1801 10^{-11}

" COMPARE WITH THE INITIAL FUNCTION", u_3, "(x)=", $1 + 0.006987409702\,x - 0.02233396848\,x^4 + 0.02787243020\,x^7$

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RECURRING LOOP No

5, "&/. ADJUSTMENT QUANTITY OF ;"5, " order WE OBTAIN : "

" FUNCTION ", u_5, "(x)=", 1

$$+ 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$$

" ALPHA COEFFICIENT," [5], "=", $-3.7900\ 10^{-14}$

" ERROR OF : ", u_5, "(x)", " AND ", u_4, "(x)", " IS :"

" ERROR ESTIMATED =, 0.

" COMPARE WITH THE INITIAL FUNCTION", u_4, "(x)=", 1

$$+ 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$$

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RECURRING LOOP No

6, "&/. ADJUSTMENT QUANTITY OF :", 6, " order WE OBTAIN : "

" FUNCTION ", u_6, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [6], "=", $9.4987\ 10^{-17}$

" ERROR OF : ", u_6, "(x)", " AND ", u_5, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_5, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

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RECURRING LOOP No

7, "&/. ADJUSTMENT QUANTITY OF :", 7, " order WE OBTAIN : "

" FUNCTION ", u_7, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [7], "=", $-2.3806\ 10^{-19}$

" ERROR OF : ", u_7, "(x)", " AND ", u_6, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_6, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

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RECURRING LOOP No

8, "&/. ADJUSTMENT QUANTITY OF :", 8, " order WE OBTAIN : "

" FUNCTION ", u_8, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [8], "=", $5.9664\ 10^{-22}$

" ERROR OF : ", u_8, "(x)", " AND ", u_7, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_7, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

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RECURRING LOOP No

9, "&/. ADJUSTMENT QUANTITY OF ;"9, " order WE
OBTAIN : "

" FUNCTION ", u_9, "(x)=", 1

$$+ 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$$

" ALPHA COEFFICIENT," [9], "=", $-1.4953\ 10^{-24}$

" ERROR OF : ", u_9, "(x)", " AND ", u_8, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_8, "(x)=", 1

$$+ 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$$

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RECURRING LOOP No

10, "&/. ADJUSTMENT QUANTITY OF ;"10, " order WE OBTAIN : "

" FUNCTION ", u_{10}, "(x)=", 1

$$+ 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$$

" ALPHA COEFFICIENT", [10], "=", $3.7476\ 10^{-27}$

" ERROR OF : ", u_{10}, "(x)", " AND ", u_9, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_9, "(x)=", 1

$$+ 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$$

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RECURRING LOOP No

11, "&/. ADJUSTMENT QUANTITY OF ;"11, " order WE OBTAIN : "

" FUNCTION ", u_{11}, "(x)=", 1

$$+ 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$$

" ALPHA COEFFICIENT", [11], "=", $-9.3925\ 10^{-30}$

" ERROR OF : ", u_{11}, "(x)", " AND ", u_{10}, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_{10}, "(x)=", $1 + 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$

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RECURRING LOOP No

12, "&/. ADJUSTMENT QUANTITY OF :", 12, " order WE OBTAIN : "

" FUNCTION ", u_{12}, "(x)=", $1 + 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$

" ALPHA COEFFICIENT", [12], "=", $2.3540\ 10^{-32}$

" ERROR OF : ", u_{12}, "(x)", " AND ", u_{11}, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_{11}, "(x)=", $1 + 0.006987409824\, x - 0.02233396875\, x^4 + 0.02787243029\, x^7$

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RECURRING LOOP No

13, "&/. ADJUSTMENT QUANTITY OF :"13, " order WE OBTAIN : "

" FUNCTION ", u_{13}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [13], "=", $-5.8997\ 10^{-35}$

" ERROR OF : ", u_{13}, "(x)", " AND ", u_{12}, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_{12}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

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RECURRING LOOP No

14, "&/. ADJUSTMENT QUANTITY OF ;"14, " order WE OBTAIN : "

" FUNCTION ", u_{14}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [14], "=", $1.4786\ 10^{-37}$

" ERROR OF : ", u_{14}, "(x)", " AND ", u_{13}, "(x)", " IS :"

" ERROR ESTIMATED =", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_{13}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

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RECURRING LOOP No

15, "&/. ADJUSTMENT QUANTITY OF ;"15, " order WE OBTAIN : "

" FUNCTION ", u_{15}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [15], "=", $-3.7058\ 10^{-40}$

" ERROR OF : ", u_{15}, "(x)", " AND ", u_{14}, "(x)", " IS :"

" ERROR ESTIMATED = ", 0.

" COMPARE WITH THE INITIAL FUNCTION", u_{14}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

"
--- \
--------------------------------"
---"

RECURRING LOOP No

16, "&/. ADJUSTMENT QUANTITY OF ;"16, " order WE OBTAIN : "

" FUNCTION ", u_{16}, "(x)=", $1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7$

" ALPHA COEFFICIENT", [16], "=", $9.2877\ 10^{-43}$

" ERROR OF : ", u_{16}, "(x)", " AND ", u_{15}, "(x)", " IS :"

" ERROR ESTIMATED = ", 0.

"-------------------------------CONCLUSION-------------------- \
----------- "
-------------------------------CONCLUSION------------------------------- "

"
THE ESTIMATED ERRORS OF '

" order ", [1], " IS :" 0.0000062884

" order ", [2], " IS :" 1.1818 10^{-8}

" order ", [3], " IS :" 3.1801 10^{-11}

" order ", [4], " IS :" 0.

" order ", [5], " IS :" 0.

" order ", [6], " IS :" 0.

" order ", [7], " IS :" 0.

" order ", [8], " IS :" 0.

" order ", [9], " IS :" 0.

" order ", [10], " IS :" 0.

" order ", [11], " IS :" 0.

" order ", [12], " IS :" 0.

" order ", [13], " IS :" 0.

" order ", [14], " IS :" 0.

" order ", [15], " IS :" 0.

```
> dothi(2,8,13,50);
```

$$\text{"GRAPHIC :"}\, u_2,\ \text{"(x) = "},\ 1 + 0.006987451495\,x - 0.02233406016\,x^4 + 0.02787245210\,x^7,\ \text{" RED "}$$

$$\text{"GRAPHIC :"}\, u_8,\ \text{"(x) = "},\ 1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7,\ \text{" YELLOW "}$$

$$\text{"GRAPHIC :"}\, u_{13},\ \text{"(x) = "},\ 1 + 0.006987409824\,x - 0.02233396875\,x^4 + 0.02787243029\,x^7,\ \text{" BLUE "}$$